King's College London

BOWLAND
MATHS

Keeping the Pizza Hot

BOWLAND
MATHS

Keeping the Pizza Hot

Case study for the Bowland Trust Project

King's College London Franklin Wilkins Building (WBW)
Waterloo Road London, Work State SE1 9NH **T** 020 7848 3855 **F** 020 7848 3182 jeremy.burke@kcl.ac.uk Work URL www.kcl.ac.uk

BOWLAND MATHS

Keeping the Pizza Hot

Contents

1. Introduction

This case study looks at the problem of maximising the market available to a Pizza shop through the ability to keep a home delivered pizza warmer for longer. The problem appeals to students because they all have had a pizza experience and so the potential 'realism' and 'utility' of the problem takes hold immediately.

The study is about building mathematical models to look at the issues which concern the owner of a Pizza Shop - *Pietro's Pizzeria*. These include (1) pizza-cooling, (2) the distance/ time travelled and (3) journeys for multiple deliveries. Mathematical modelling is recognised as important in the Key Stage 3 National Curriculum which says:

> Mathematics equips students with uniquely powerful ways to describe, analyse and change the world

Here we are concerned to describe and analyse the issues concerning home delivery and to make suggestions/recommendation which could change the organisation of the business.

This case study is concerned not only to introduce some interesting mathematics at Key Stage 3 but also the idea about the process mathematical modelling needing to include an awareness of the originating context and the assumptions and simplifications made when constructing a mathematical problem.

This case study is aimed at KS3 and could be used with years 7, 8 or 9 although year 8 might be best. The back story of pizza is available to all students and because the mathematics content will probably be new to all then all should be able to engage with it. There is an assumption that students will have met linear graphs prior to engaging with this work, and plotting points on a Cartesian grid.

The important point is for teachers to say less and listen more. The students can get very excited during these lessons and all want to speak at once. Organising these discussions can be tricky so we recommend they work in small groups of 3 or 4, who can then come together as a whole class for reporting-back purposes. You might wish to handle the discussions by writing up points and questions as they arise in groups. This allows other groups to see these and incorporate them into their discussions. A plenary will allow you to deal with points and pull the discussion together.

2. Overview

Keeping the pizza hot is a perennial problem for home delivery food businesses. They need to maximise their potential business by having wide delivery areas, but they also need to keep the pizza attractive when it arrives. This problem provides an opportunity for students to address the questions:

- what are the best materials for packaging the delivery of food
- what is the longest reasonable travel time (before the pizza becomes too cold)

This project involves a number of parts which lead students to move from a practical problem of a cooling pizza to a mathematical representation, a cooling curve. This is a big step and inducts students into the potential of mathematical applications. It demonstrates how mathematics might underpin scientific enquiry.

The project starts with a short presentation sequence which sets up the problem. The pizza shop shop owner needs to know about the temperature drop in order to work out the catchment area for the business. Students are then invited to look at how the problem could be addressed and they will come up with the idea of a 'fair test'. Here there are two possibilities. Either links can be made with Science and students can carry out an experiment, or alternatively a video clip shows students undertaking the experiment and gives details of the results.

The students then move to a mathematical phase where they have to fit an appropriate function to the raw data. Cooling does not happen in a 'neat and simple' way, but a mathematical description of the cooling rate is possible by 'fitting' a function to the stream of temperature readings as the warm material cools.

Having found an appropriate function, students are then asked to address the problem: which packaging is best? There are a number of key questions:

- what is a satisfactory temperature to deliver a pizza?
- How long will it take for a pizza to reach that temperature in each packaging type?
- What is the potential market available with a given delivery time?

Finally, students can construct a presentation or a display. This could take the format of a report, an 'article' for a trade journal (*Pizza Monthly*), a powerpoint presentation, a webpage perhaps include suitable photographs, diagrams and images

3. Mathematical Content

This case study is ambitious in that it expects that students will be able to engage with simple mathematical modelling, and even simple modelling is challenging in terms of KS3 mathematics. A number of issues arise out of the home delivery question, 'How do we delivery a hot pizza?'

Pizza cooling

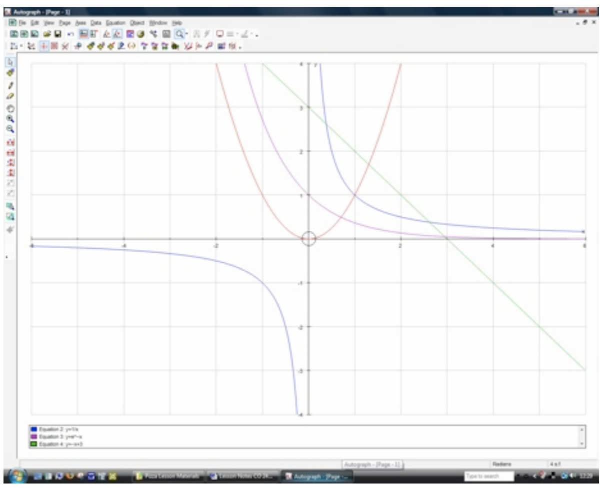
The cooling problem expects students to be able to be introduced to a range of functions and explore some of their attributes – quadratic, inverse, and exponential functions. Looking at data on cooling suggests that a straight line should be a good fit, and indeed it is for the first few readings. However the use of linear functions to model cooling will lead to a prediction of the temperature of the pizza falling to freezing – suggesting the self freezing pizza. Students have no difficulty in recognising that this is not sensible. If a curved function is needed then typically a quadratic is the first one that students meet. Again a quadratic will give a good initial fit. However, here after a while, it would predict that the pizza begins to heat up and students realise that they haven't experienced a 'self heating' pizza before. So the hunt is on for a suitable shaped function. An inverse function doesn't so it, but an exponential function does.

Distance/time travelled

Another issue is the distance that a moped rider can reasonably reach in a given time. Here the students can work out potential market area, ie radius and area, and then refine this to a more complex view of a map, with more limited routes from shop to home(s).

Journeys

A more economic form of delivery might be to group drop off points into one continuous journey. Here the model would be concerned with a complex grouping of distance/time measurements, drop off times, allowing for houses and blocks of flats, allowing for road restrictions, one way systems and so on. Using a local map this could be reconfigured into a network of possible journeys - perhaps something like the London Tube Map.



4. Organisation and Pedagogy

Organising discussions … saying less, listening more

The students can get very excited during these lessons and all want to speak at once… and so do teachers. If you look at the first video clip in the presentation we have included a clip of some teachers themselves working on this project. They are quite 'buzzy', making conjectures and trying to get a 'feel' for the data. Then suddenly one teacher says, "During this time when the kids are just doing this lots of restlessness would occur". Here the teacher has flipped from student-mode to teacher-mode. Up to this point the teachers were engaged, even quite excited. We have found that students are also similarly engaged … given half a chance.

We recognise that organising these discussions can be tricky. The students do generally generate good questions and ideas about the pizza problem and this allows you to concentrate on encouraging good discussions.

We recommend that you use a mix small group and whole class discussion. Small group discussion gives the students time to express and develop their ideas. It also gives you the chance to listen to the students and to begin planning the whole class discussion: "That's interesting. I'd like you to share that with the class."

One frequently effective technique is to say very little and to record students' responses on the board. This allows you to listen to (and think about) what the students say, it helps the students remember all the things that have been said and it enables you to concentrate on making sure that all the students have an opportunity to contribute.

Write up incorrect and partially correct ideas as well as correct ones. This encourages the students to make hunches, to agree and disagree with each other and to challenge and improve upon each others' ideas: "Anya says that it's not a straight line. Do you agree?" … "Jacob thinks the crow flies distance isn't perfect but that it's good enough. Vicky disagrees and says we should measure exactly. What do you think?" … "Well, if you're not sure, you could ask Chris a question."

If the students have difficulty coming up with ideas, ask them to talk about the problem in pairs or give them some individual "think time". Suggest they write down their ideas on a piece of paper or a mini-whiteboard to use as an aide-memoire for the discussion.

Encouraging students to be mathematical

The strategy of saying less and listening more allows you to make more planned and carefully thought out interventions. We have found one crucial intervention to be at the point when students start talking about the shape of the cooling curve. Generally, they begin by thinking of it as a straight line. Some teachers have said, “So it looks straight. Is it?”, whilst others have simply made a straight rule available without saying anything. Another technique is simply to make a statement, e.g. “It looks straight to me” – allowing students to correct or amend a teacher’s statement is very powerful.

BOWLAND MATHS

5. Resources

The supplied teaching resources consist of:

- A 2 page introduction to the project.
- A PowerPoint presentation with slides to introduce the problem to students and two further slides with links to the supporting video sequences, data files and software.
- Video sequences showing the pizza cooling experiment, one specific cooling example in detail, the taste test experiment and 5 video sequences showing the lessons in a classroom setting.
- Sets of data files for use in (a) *Equator* and (b) A graph plotter (e.g. *Autograph* or Graph)
- 4 Student worksheets.
- The software programme: *Equator*.

You will need access to all of these.

Minimum

This represents the minimum additional equipment required to do this activity

- Digital projector for PowerPoint and Google Maps (or even Google Earth)
- Teacher access to a computer connected to a projector running graph drawing software such as *Autograph* or Graph.
- Student access to computers.

Using the PowerPoint Presentation

Slide 6 in the presentation is designed to engage students in the experiment, where it has proved impossible to set it up practically.

- The first video clip shows a group conducting the experiment, heating and cooling pizzas, observing the graphs, talking about them, changing the packaging and trying again.

- The second clip shows a run through of one complete cooling sequence from which students can collect data on worksheet 1.

- The third clip shows the taste test experiment being conducted, in which a student cools a pizza, checking for the lowest temperature at which the pizza would still be acceptable to deliver.

- There follows a link to a complete set of data in Excel format. This will open to a page with data configured to be copied and pasted into the *Equator* software. There is a second tab on the spreadsheet containing the full data sets. There is data for 4 different types of packaging.

- Finally there is a link which will launch the *Equator* software. Look at the software guide in appendix E for tips on using *Equator*.

Experiment

We would strongly recommend that the experiment is done in class. The best model would have groups of students conducting the experiment for themselves, although a single experiment conducted by the teacher has proved effective.

- You will need a microwave oven in the room. We have found that by borrowing (e.g. from the staffroom or science or food technology departments), that 3 ovens for a group of 30 grouped in fives, works well.

For each experiment group you will need:

- Temperature probes and software. The most reliable and easiest to use are Vernier Go Temp probes which come with Logger Lite software.
  (These are available from: https://www.studentcalculators.co.uk/acatalog/test_CO.html).

- Mini Pizza (We found that Chicago Town mini deep dish pizzas were very effective).
- Different types of packaging as discussed: we used aluminium foil, padded posting (jiffy) bags and cardboard packing boxes.

Plenary

To manage a plenary discussion using Google Maps, you will need a reliable internet connection on the teacher computer.

6. Teacher notes (a)

This represents a possible 4 lesson sequence. However, this is squeezing the project very tightly into the time allowance. If we consider these as four parts, or four stages, then more time might be added to each stage as needed. Each Lesson description ends with a recommendation about time allowance.

Lesson 1: The Problem and the Experiment.

Part 1: Introduce the problem using the 'Pizza intro' presentation.

- Students discuss in small groups:
- What is an acceptable temperature for delivering a pizza?
- How far away from the shop can the business do deliveries?
- How could they reach more customers?
- Scooters do not drive ‘as the crow flies’ so actual routes could be different.
- Most deliveries involve more than one pizza. What would be an appropriate route plan for making multiple deliveries making sure that the first one doesn’t arrive too hot!
- What information do we need to examine these issues?

Whole class share their conclusions. (A record will need to be kept to inform the conclusions in the final lesson).

Part 2: Experiment to find the best packaging for delivering pizzas to maximise the number of customer who can successfully be reached.

See 'the pizza cooling experiment' in the next section (page 9).

Time Allowance: Students can take quite a time to do the experiment, so consider whether more time is needed here if you are doing the activity. It has certainly paid off in terms of students’ connexion with the problem and drive to see it through to the end.

Lesson 2: How long does it take until the pizza is too cold?

Part 1: Review the experimental data from lesson 1. Whole class agreement on the range of acceptable temperatures for a pizza needs to be reached. The experimental data is not enough to answer the question, so a graph is needed. Students work in pairs to construct graphs of the data by hand on graph paper.. Whole class agreement on the size and scaling of the axes is important. If we want to know when the pizza is too cold, we will need to extend our horizontal (time) axis beyond the data far enough that we reach that time. Also, the vertical axis should extend to a little below zero, to see the effect of conclusions.

Part 2: Students use their graphs to predict the times within which the pizza is acceptable. The graphs will need to be extrapolated somehow so students should describe their reasoning.

This is repeated for the other two data sets (for the different pizza packaging).

Part 3: "OK, but how long will it take to reach room temperature"

We could draw another graph.

What would be the problem with that? (e.g. tiny data set on a massive long graph will get increasingly inaccurate).

We could find an equation for the line we put on the graph.

[Whole class discussion, using *Autograph* or *Equator* with the data from one of the experiments].

Try a straight line – it's a scatter plot, so a line of best fit seems logical.

Fit a straight line to the data, discussing the variation in m and c as you change the line to improve the fit. When all are agreed that the fit is good, separate the formula from the graph. Substitute values into the function to check that they accord with the values on the graph. Find the times for the greatest and least acceptable temperatures.

Answer the question: when will it reach room temperature?

Time Allowance: Students might need to take some time to feel comfortable with *Equator* and enter their data. We have found that the conversation can get very intense as students discuss whether a straight line is good enough, and whether the data plotted is linear or not. You may not wish to truncate any discussion too harshly.

Lesson 3: Improving on a linear model.

Part 1: What happens to the temperature of a pizza as it cools in the long term?

What's wrong with using a straight line as a model for the cooling of a pizza?

[e.g. When does the model say it will reach zero? Does this actually happen to pizzas?]

Part 2: Students work in pairs using the *Equator* software to find a better model for the data. They should try out all of the possible function types, try to find a good fit for the data, then describe any long term problems with the model. They need to find the best model (which fits the data and works well in the long term). They should use the function for their model to find the times for the greatest and least acceptable temperatures. This should be repeated for the other two data sets. Conclusions should be reached about which is the best packaging.

Lesson 4: A report for the pizza business.

The key issues for the pizza business that were discussed in lesson1 can be reviewed. The central issue is how far the pizza can be delivered at an acceptable temperature. A whole class discussion can consider how this can be calculated knowing the time of cooling. [e.g. A scooter travelling at a certain average speed, can reach a certain radius from the shop in the given time].

Students now have enough information to write their report to the pizza business. Central to the report will be:

- a description of the different types of packaging choosing the best one.

- calculations showing the distances that can be reached.

Finally, students should consider what the response of the Pizza shop owner might be. Does the mathematical problem actually get at all of the things that will effect the decision at the level of the business?

It would be best if the task were extended according to the considerations that students made when addressing the problem in lesson 1. Here are some possible issues that could be addressed:

- why do pizza shops actually use boxes with holes in them? Design an optimal package for a delivery pizza.
- density of housing can suggest different target customer bases, how many customers can be reached, what is the probability of a sale on a given day, turnover, costs, profits ...
- competition: what is the density of takeaway pizza shops? What is the density of other takeaway food outlets: fish & chips, kebabs, Indian and Chinese restaurants etc.

Teacher Notes (b):

The Pizza Cooling Experiment

Many teachers have actually carried out the pizza cooling experiment and found that this aids students' engagement with the problem. You may want to do this with colleagues from science.

Two models have been trialled successfully:

a) The teacher managing the experiment from the front of the class as students actively engage with the unfolding data.

b) 6 groups each of 5 students conducting the experiment themselves.

Equipment

- Microwave oven (one is enough for groupwork or teacher led models).
- Data logging equipment (see below)
- Mini pizzas (microwave mini deep pan pizzas worked well)
- Packaging (e.g.)
  - delivery style box self cut from old cardboard boxes
  - silver baking foil
  - A5+ padded posting bags
- Plate (for heating the pizza on and safely removing it from the oven).
- Oven gloves, tongs, heat resistant gloves or similar for removing the pizzas safely.

Data Logging Equipment

Most science departments will have data logging equipment. This normal consists of software, an interface box to connect to the computer on which the software is loaded and a temperature probe. It is strongly advised that the equipment is set up and tested in advance as especially older systems can be unreliable.

We had most success using a system made by Vernier which consists of an inexpensive temperature probe which plugs directly into the USB port of a computer and a simple software programme, which, when installed, will load automatically when the probe is connected. Projecting a view of the data logging graph onto a white board was very successful for the teacher led model. With groups, each groups will need a laptop or similar to watch the graph and gather the data.

Equally successful is to use TI-83 or 84 graphical calculators with Easy-Temp probes running Easy-Data software. The taste test experiment (see below) was conducted using a TI-84 running as a digital thermometer.

Note that the probes take about 10 seconds to reach the temperature of the pizza from room temperature, hence the temperature of the pizza is measured as rising in this time. When the data is copied and pasted from the data logger, it is best to start from the point at which the highest temperature reading has been reached. The supplied data is compiled in this way.

Managing the Experiment

As the experiment unfolds, it is important to engage students in their improving capacity to use the data to make predictions.

Most schools have started with a control experiment letting the pizza cool with no packaging. The experiment is then repeated with different types of packaging.

1. Follow the instructions on the package to heat the pizza, on a plate.

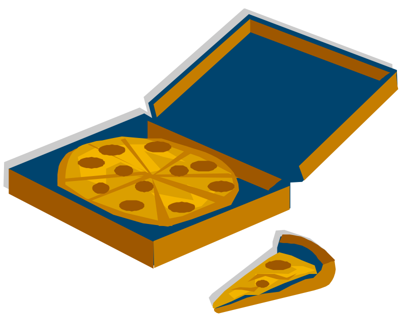
2. Remove the pizza put as quickly as possible into the packaging, insert the probe (into the side of a deep pan pizza across and hence through the base, worked well).

3. Start the data logger.

Check the temperature at appropriate intervals, starting with an initial reading (and time = 0). Most school found 10 readings at intervals of one minute was appropriate for hand drawn graphs. (The logging equipment is generating data at very small intervals and hence generates a much smoother graph initially). Some schools found that having students make a table in their books showing times t=0 up to t=10 ready to record the temperature was very engaging. The teacher or a member of the group would read off the temperature as each minute ended and the group monitor would write it into the table. As the experiment continued students can predict the temperature in at the end of the next minute, and at the end of the experiment (and in one hours time, and tomorrow). They should write down their predictions and explain on what basis they were made. As the experiment unfolds, they should update their predictions for the longer term.

Worksheet 1 has been provided to support his process.

The Taste Test Experiment



A key feature of the analysis is to know at what temperature a pizza could not acceptably be delivered. A possible experiment to determine this point is as follows:

- Heat a pizza as usual in the microwave. Unwrapped, but on a plate for safety.
- At the end of heating time, insert a probe and set the logging software running.
- Read the temperature and taste the pizza at various temperatures. [Do NOT taste the pizza above 75° for safety reasons!]
- Note the point at which you consider the pizza to be unacceptably cold. The least acceptable temperature would then be the reading before this one. [readings taken at 74°, 72°, 70°, 68°, etc. downwards would be sufficient].

There is footage of this experiment being carried out on the presentation.

Our results showed that 62° was the lowest acceptable temperature.

Teacher notes (c)

Using ICT

This project offers the possibility of using ICT in a number of ways: for data logging; for using an applet (*Equator*) to get the 'feel' of different functions; for using a graphing programme (such as *Autograph* or *Omnigraph* etc); for using the internet to find out about pizza delivery; for producing reports and presentations.

Clearly how this can be integrated into lessons is dependent on access to computers. In this sequence of lessons minimally students would be expected to have access to computers for lessons/parts 2 and 3. This is so they can explore the various functions making use of the graphing software.

If students are going to use *Equator* in the computer room, then it will need to be installed on each machine – it is very small and will take only a moment to load, or download from a school webpage. The applet is written in Java script so please make sure that the latest version of Java is available on your computer room machines. Java can be downloaded for free, as your network manager will probably know.

It is important to 'play' with the programs before using with students – so try entering data and seeing how the graph lines can be manipulated. If you are going to use a graph plotter, then check that you know where data can be entered so the graph can be 'fine-tuned'.

Teacher notes (d)

Writing a report

Since this is an exercise in mathematical modelling then we would expect students to write a report for the Pizza shop owner to use to help him make a decision, or to write a report from the point of view of the Pizza shop owner.

The report could take a number of formats:

- Write a report in the form of a mathematical discussion about the model, the assumptions made, the choice of function and the predicted cooling time. This could finish with a recommendation to the Pizza shop owner.
- Write a report in the form of a journal article for a trade magazine, perhaps, “Keeping the Pizza Hot! How Pietro solved his delivery problems”
- Write the details of the problem and how it was solved and how it would be judged by the Pizza shop owner as a series of PowerPoint slides. This would then make a possible format for presentation at a school assembly or to parents attending a Parents’ Evening.
- Design a web page explaining mathematical modelling giving pizza and cooling curves as an example.

The assessment of the project might be against a range of criteria:

1. Do students’ reports, in whatever format, indicate an understanding of the National Curriculum PoS listed in Appendix A?

2. Have students developed further their ability at manipulating graphs and understanding the transforming effects of coefficients and powers?

3. Have students developed an understanding of mathematical modelling as a way of describing an event as a mathematical problem? Have they recognised what is meant by “assumptions” in the modelling process?

4. Did they appear to enjoy the project? Do they feel more positive about engaging in mathematics?

Each of these elements tend to lead to qualitative judgments, but these are important to balance against quantitative assessment at other times.

7. Supporting worksheets

There are four worksheets here:

1. A support sheet for students to collect data at minute by minute intervals while doing the pizza experiment. They are encouraged to make regular predictions and to improve the quality of their predictions as the experiment progresses. Each student will need one sheet for each experiment i.e. 3 or 4 each.
2. A sheet to support students plotting graphs and finding lines of best fit. The program *Equator* or a graph plotter will help students fit a graph line by eye, but using the graph paper helps students get a better sense of where data is plotted and why.
3. A sheet to log the results from experimenting with *Equator* to find better models of the cooling data.
4. A worksheet is to support final lesson summaries. Here students are encouraged to look back at the questions they asked at the beginning of the case study and to address these. This marks the shift from looking at the mathematics used for modelling to a judgment about how the mathematics informs a judgment back at the Pizza shop.

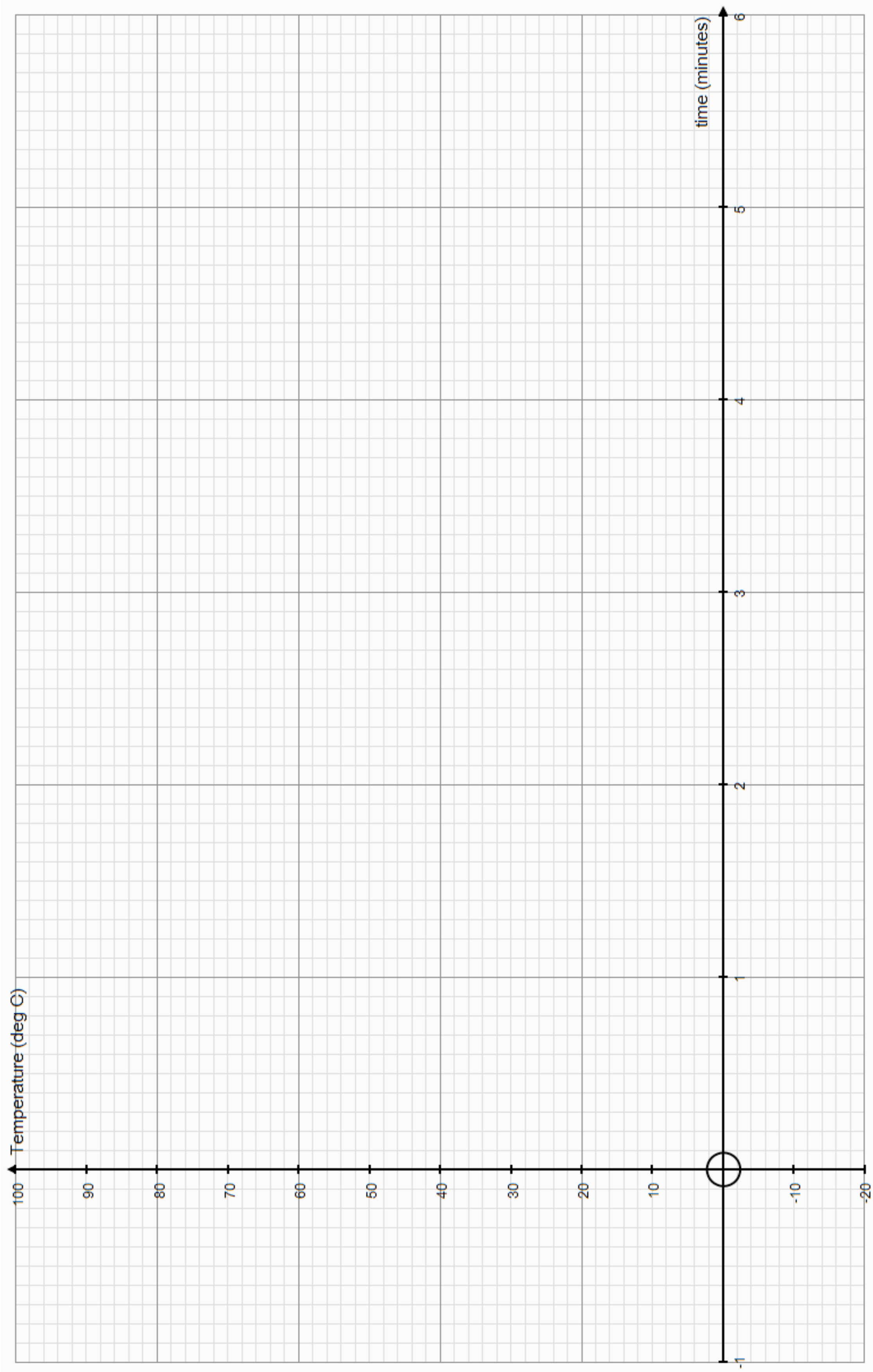
Worksheet 1: Experimental Data Collection and Prediction Sheet

Make predictions:

1. After every reading predict the temperature at the next minute.
2. After 2 minutes predict the temperature at 5 minutes.
3. After 5 minutes predict the temperature at 10 minutes.
4. After 10 minutes predict the temperature at 30 minutes, 120 minutes and 24 hours.

Time (T) (minutes)	Temperature (t) (deg. C)	Prediction	How did you make your prediction?
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
30			
120			
24 hours			

Worksheet 2: Plotting the data



Worksheet 3: Graph Log

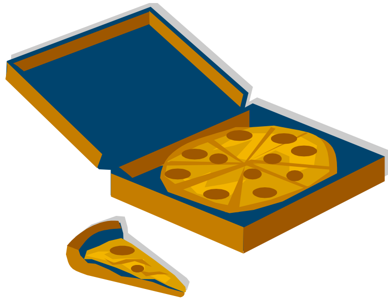
This sheet can be used to record the observations made using *Equator*

Sketch of graph	**Function (with an example calculation)**
Read off from the graph: The temperature after 1 hour: The time taken to reach the minimum acceptable delivery temperature:	**My comments on how well this function fits the temperature in the short, the medium and the long term:**

Worksheet 4.

The Pizza Project

1. Write a short intro:
    a. What problem are you trying to solve
    b. What are the factors that affect the problem
    c. What experiment did we do



2. Complete 3 graphs:
    a. No insulation
    b. Foil
    c. Jiffy bag
3. On each graph add a suitable line to help you predict the temperature at future times.
4. Use your lines to predict what the temperature will be after 45mins.
5. What is the problem with your straight line for predicting?
6. What would be a more suitable type of line?

Consider the pizza shop owner:

7. The pizza shop owner wants the pizzas to be at least 65° when they arrive. How long does that give for delivery for the 3 different conditions?
8. The drivers can only drive at 30 miles per hour, how far will they be able to deliver?
9. Complete your write up including your intro, graphs, answers to the questions above and conclusions.

10. Project to be handed in on ____________________

8. Lesson Plans

A set of structured lesson plans that could be used for the suggested sequence.

PIZZA COOLING – LESSON 1 (60 minutes)

TiMES	POINTS	NOTES
9 min	**Introduction** Power point display of 'comic' story about the Pizza that arrived … too cold! Establish the question: How can the owner of the Pizza Shop ensure that the customers receive hot pizzas?	**Equipment Required:** PowerPoint
15 min	**Discussion – How can the pizza be delivered hot?** This is an 'orientation' discussion which allows students to get to grips with the problem. Issues that will arise: starting temperature; packaging; time taken to deliver; how far can a moped reach in the time. Focus the discussion: what would count as a fair test?	
Here decide whether students will (i) do the experiment and generate their own data or (ii) use the video and data provided		
30 min **(i)**	**(i) The experiment and the data** In groups students take a heated pizza, wrap it in an insulation material, insert a temperature probe, watch what happens as it cools. Here students will be encouraged to ask questions: What will happen in the next minute? What will be the temperature in the next 5 minutes? How cool will the pizza eventually become? NB *If students are going to do this themselves then it may well take longer than 30 minutes. You may consider setting up a lesson with the Science department or with Food Technology to make curriculum connection and to utilise other members of staff.*	Microwave oven(s) Data logging equipment Mini pizzas Packaging Plate Oven gloves, tongs, heat resistant gloves or similar Worksheet 1 (4 copies per student)
30 min **(ii)**	**The experiment and the data (ii)** Show video of carrying out the experiment. Students ask the predictive questions as the data appears in the video. Show the Taste Test video clip.	Video clip on presentation: Testing the packaging The Taste Test
6 min	**Recap.** the main points about the problem and how researching packaging may help address the problem. The next lesson will look in more detail at the analysis of the temperature data.	

Time

If carrying out the experiment then perhaps a longer time needs to be allowed – either a double period, or a 'special' lesson which runs into lunch time or after school might be appropriate. A project might be negotiable with other subjects to give up one period to help facilitate a double period of maths.

PIZZA COOLING – LESSON 2 (60 minutes)

TiMES	POINTS	NOTES
2 min	**Introduction** Recap of what we did in the previous lesson.	**Equipment Required:** *Equator* and/or graph plotting software on teacher machine Data Graph Paper OR Worksheet 2 (1 copy per student)
8 min	**Discussion – Fitting a function so that we can predict for times in the future** Putting the data back up on the board, explain that our experiment gave us an idea of what the pizza cooling looked like but we need to be able to predict for future times. What could we do? Fit a line to the bottom section of the data, ask the students what they think the temperature will be in 1 hour, 12 hours. So the model works well, what does this mean for the shop owner?	
40 min	**Activity – Looking at their own data (i)** Students to plot their data and extend a line of best fit in order to see how long the pizza will stay 'warm enough'. **Homework** Finish graphs for 2 insulation types and make notes on what we have done so far in order for them to write up over the weekend.	
10 min	**Discussion – Limitations of the model?** What would their line tell them that the temperature would be in 2 days time? So what type of line might we try next…	

Time

If the students are using *Equator* then they might need some practice at moving the graphs around. It might also take some time to enter the data. These issue depend on the class, but it is worth considering whether some 'expansion' time is needed in order to complete the project.

PIZZA COOLING – LESSON 3 (60 minutes)

TiMES	POINTS	NOTES
2 min	**Introduction** Recap of what we did in the previous lesson. Used *Equator* and found that a straight line predicted the self-freezing pizza. Also found at which point the straight line stopped being a 'good fit'	**Equipment Required:** *Equator* software with laptops/ computer room Data Graph Paper and/or Worksheet 3 (4 copies per student or per pair)
8 min	**Discussion – Fitting a function so that we can predict for times in the future** The last lesson showed that a straight line was a good fit for the initial cooling data, although the data always 'bent away' from the straight line. It was also found that the straight line predicted temperatures falling to zero – the self freezing pizza.	
40 min	**Activity – Looking at their own data (i)** This lesson looks at alternative functions, using *Equator* again. Students should try and fit a curve to the data form the choice offered. When they have what appears to be a good fit, they should zoom out their graphs and see what the prediction would be. **Homework** Finish graphs for 2 insulation types and make notes on what we have done so far in order for them to write up over the weekend.	
10 min	**Discussion – Limitations of the model?** What does the quadratic equation predict? A self heating pizza? What does the exponential equation predict?	

Time

This lesson has worked well in the 60 minute time. However, there is the potential for some serious discussion about the functions, their 'shapes' and how they can be 'flattened' and transformed

PIZZA COOLING – LESSON 4 (60 minutes)

TiMES	POINTS	NOTES
2 min	Introduction Recap of what we did in the previous lesson.	**Equipment Required:** Teacher computer running Google maps on an internet connection. Worksheet 4 (1 copy per student)
8 min	Discussion – Fitting a function so that we can predict for times in the future Look at the outcomes of the previous three lessons, and remind students that the problem was to consider how the Pizza Shop could expand its customer base. The modelling of pizza cooling allows us to predict how long it will stay warm enough to be acceptable to customers on delivery. Since mathematical modelling is being used, then it is important to list the significant assumptions that have been made in constructing the model. assumes market circular around shop assumes bikes travel at constant speed assumes one pizza per journey assumes 'straight line' journeys. These assumptions permit a relation to distance travelled and market size – basically a circle round the shop, distance travelled is the radius so the market increases for each increase in d Ò πd^2.	
40 min	**Activity – Looking at their own data (i)** Using the worksheet for Lesson 4 as a guide students can write up their project	
10 min	**Discussion – Limitations of the model?** Check that work is completed and if needed to be finished for homework	

Time

The final part of the sequence is about making sense of the mathematics on the one hand, and then 'interpreting' the mathematics in terms of the original problem

Appendix A: National Curriculum Content References

(2007 National Curriculum http://curriculum.qca.org.uk)

Key Concepts: all of the key concepts can be addressed through this work, however the following are of particular relevance:

1.1 Competence

a. Applying suitable mathematics accurately within the classroom and beyond.

b. Communicating mathematics effectively.

1.4 Critical understanding

a. Knowing that mathematics is essentially abstract and can be used to model, interpret or represent situations.

b. Recognising the limitations and scope of a model or representation

Key Processes: the following are particularly addressed:

2.1 Representing

a. identify the mathematical aspects of a situation or problem

b. choose between representations

c. simplify the situation or problem in order to represent it mathematically, using appropriate variables, symbols, diagrams and models

d. select mathematical information, methods and tools to use.

2.2 Analysing

a. make connections within mathematics

b. make and begin to justify conjectures and generalisations, considering special cases and counterexamples

c. explore the effects of varying values and look for invariance and covariance

d. take account of feedback and learn from mistakes

e. appreciate that there are a number of different techniques that can be used to analyse a situation

2.3 Interpreting and evaluating

a. form convincing arguments based on findings and make general statements

b. consider the assumptions made and the appropriateness and accuracy of results and conclusions

c. relate findings to the original context, identifying whether they support or refute conjectures

2.4 Communicating and reflecting

a. communicate findings effectively

b. engage in mathematical discussion of results

Range and Scope: This work will provide opportunities to use mathematics from a range of areas, the following will be most significant:

3.1 Number and algebra

a. linear equations, formulae

b. analytical, graphical and numerical methods for solving equations

c. polynomial graphs

3.2 Geometry and measures

a. units, compound measures and conversions

Curriculum Opportunities: the nature of this task means that it is particularly suitable for addressing all of the curriculum opportunities statements.

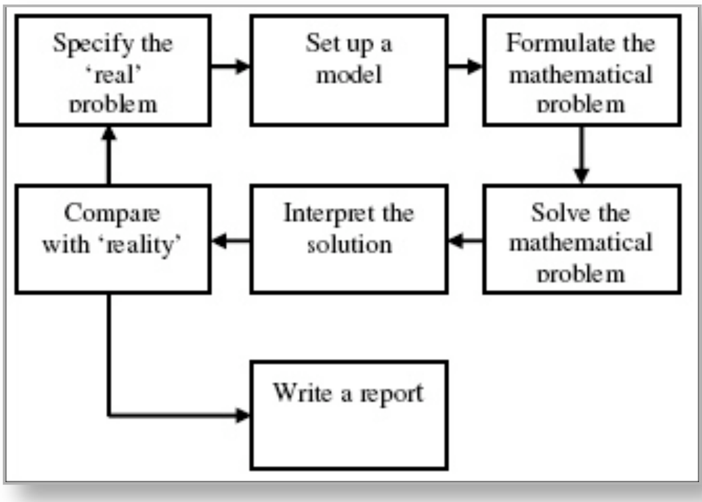
Appendix B: Mathematical Modelling

Problem

The height of a door is 6.8 units more than its width and the distance between the (opposite) corners is 100, find the dimensions of the door.

This problem comes from a Chinese text, the Jiuzhang Suanshu (meaning: 'nine chapters on the Mathematical art') based on work probably dating from around 600 BC.

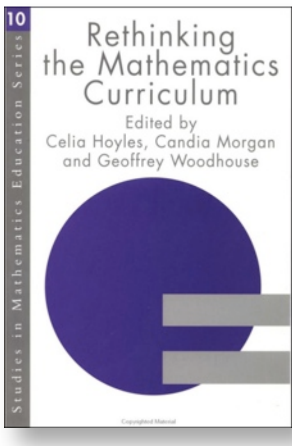
The solving of practical problems using mathematical techniques has a long history. To solve this problem first we must mathematise it. This involves creating an equation with features that represent aspects of the practical situation. In this case Pythagoras' theorem works effectively for right-angled triangles. We assume that the door is a rectangle (although this is not stated) and that therefore 'the distance between the opposite corners' is the hypotenuse of a right-angled triangle. We then manipulate the mathematical equation to produce a



mathematical solution. We check this solution against the practical situation. If necessary we go back and look again at our assumptions and make appropriate modifications to our mathematics before repeating the loop again.

The Modelling Cycle

Carolyn English, 1999, 'Modelling for the new millennium' in Celia Hoyles et al (eds) Rethinking the Mathematics Curriculum, Falmer Press: London



The mathematised version of the practical situation is a model. It is not the practical situation itself. Because mathematics has built up a vast array of tools for manipulating mathematical statements, it is a productive mechanism for modelling. It is not the only way. Visitors to the mathematics section of the science museum in London will have seen a large model of the economy built out of pipes through which flows water that is controlled by the relationships between the various economic elements.

It would not be unreasonable to suggest that modelling and applied mathematics are synonyms – in order to solve a non-mathematical problem with mathematical tools requires formulating a mathematical model of the situation. The National Curriculum does not explicitly mention modelling except at KS4 Higher 6d: "….. interpret graphs modelling real situations". However, all of the tools that are being demonstrated and practiced throughout are made purposeful by generating mathematical models and manipulating them. In fact the data handling section of the national curriculum has been re-organised to take full account of the need for modelling statistical situations.

Appendix C: A mathematical development of the pizza problem

There are a range of issues that can be dealt with by students in dealing with the problem of delivering pizzas in appropriate packaging at and appropriate temperature. However, there is a core problem which is to find an effective model for calculating the temperature of a cooling pizza in a given packaging over time. It is important to retain flexibility in accommodating students thoughts and inputs to the discussion, however, there is a general development which students are likely to follow:

1. Anecdotal discussion of trends using the predictions made when collecting the data generate a range of issues. Generally students will look at the rate of cooling (by comparing the changes from the 0th to 1st, 1st to 2nd, 2nd to 3rd etc. seconds) and assume that it will continue i.e. A linear model. Others may notice the rate is decreasing and make some accommodation. Notably many students will have views on what happens in the longer term e.g. “It will go down to zero”, will be improved to “it will go down to room temperature”.

2. When graphs have been drawn students can sketch how the graph would extend and make predictions based on their sketches. However, these don't give predictive power over what cannot be drawn on the axes.

3. Students are unlikely to have developed a relationship between the graph and the function of anything other than straight line graphs, hence a **line** of best fit is a logical starting point for many. The line can be fitted to the data using *Equator*. Students can test the predictive power of the function (which is poor in the first few minutes but good in the next many minutes). They need to discuss the reasonableness of the predictions in the longer term ... (an auto-freezing pizza runs counter to most student's experience!)

4. When students experiment with *Equator* they are likely to find that a quadratic function fits the data very closely for the first ten minutes (and apparently beyond) however, in the longer term an auto-reheating pizza does not fit their experience. An exponential function fits the data and tends towards a limit as does the experiential understanding that the pizza will not get colder than room temperature. Different functions can then be found for other packaging. The critical feature of the development is for students to see a need to find a way to accurately predict the temperature over time and at given points in time. A formula is the best method for achieving this as it is much more flexible than a graph. Hence the ability to make good predictions is central to the mathematical development.

Appendix D: Drawing Conclusions

The purpose of the model is to give better predictive power over the real world situation. It is very important that students conclude their work by making recommendations to the owner of the pizza shop about the distances that s/he will be able to deliver their pizzas.

If time permits, students can work in small groups to prepare a presentation. Posters, PowerPoint presentations and mini-plays have all been successful. However, our experience suggests that leaving students to frame the report themselves keeping note of the brief, is important. It is easy to see the presentation as an end in itself and guide students to a standard document but this would not accord with the spirit of the work.

The project would make an interesting topic for a year group assembly, the school newspaper, the school website, or for a powerpoint presentation at Parents' Evening - "Look, Mum, at what I've done in maths! Pizza *and* exponential functions!"

The key features could include:

- A discussion as to how conclusions were reached about making predictions for the time taken to reach the lowest acceptable temperature.
- Calculations showing how long it would take to reach this point.
- A map of the locality (e.g. From Google maps http://maps.google.com/ or similar). Students can place the shop and draw circles to show the distance that the acceptable pizza can reach.
- The emphasis should be on describing how decisions were made and what they mean rather than on merely presenting the data and the graph and function outcomes.
- Being explicit about the assumptions made in coming to a conclusion is also an important part of mathematical modelling, so the students could list the assumptions made here: eg a small pizza will cool at the same rate as a larger or thicker one; that all pizze are packed at the same starting temperature; that scooters will usually travel close to the average speed (ie small standard deviation), and so on.

Appendix E: Technical guides

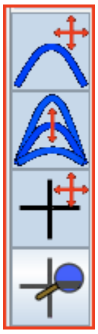
- Using the *Equator* software
- Developing the argument with Graph
- Using Vernier probes and Go! Software

Equator

Equator is a clever little program which has been written for this project. It allows students to change the 'shape' of a curve by dragging a point, and is an in-between stage between a pencil and paper fitting-by-eye and using a full blown graphing package.

The applet allows students to enter data which will be plotted on the grid; select a function type (linear, 'inverse', quadratic, exponential); move the graph up and down or side to side; flatten or make more steep by dragging. The thickness and colour of the line can be altered to taste.

The *Equator* Toolbox



Translate the graph. Click the tool, then click and drag on the graph line.

Reshape the graph. Click the tool then click and drag and different points around the graph to reshape it. The effect is different for different equation types.

Scroll the axes. Click and drag to change the part of the axes on view.

Zoom In. Click and drag a box around an area to be zoomed in on.

Zoom Out. Click in any white space on the graph to zoom out.

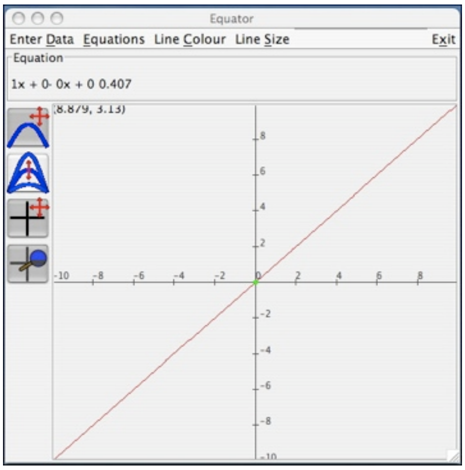
Reset Graph. Use this to return to the default axes. Especially useful if you have zoomed too much

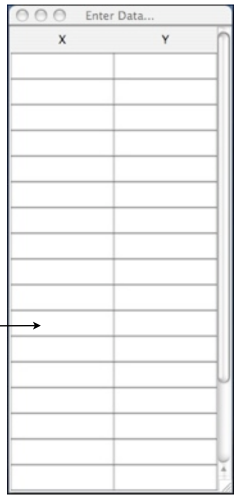
Entering data

Data from the experiment can be entered by hand or cut and pasted from a spreadsheet.

Click on the

menu item **Enter Data**





and an ***xy*** table pops up

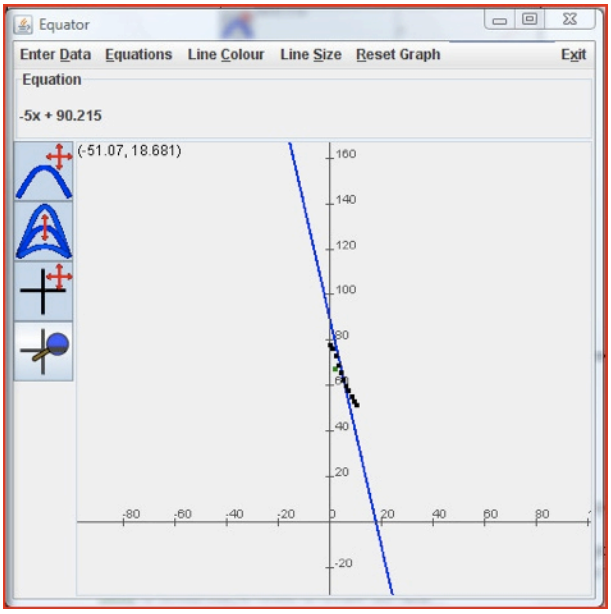
Data can be typed in or pasted. Click on the Update button at the bottom of the list to enter data into the graph (you may need to scroll down to see it).

Students can either type their minute-by-minute data gathered onto worksheet one or they can copy and paste from one of the supplied data sets.

Initially the data might not be visible on the screen. Here you can point out to students that the axes might not be in range of the data in which case they will need to zoom out in order to locate the data.

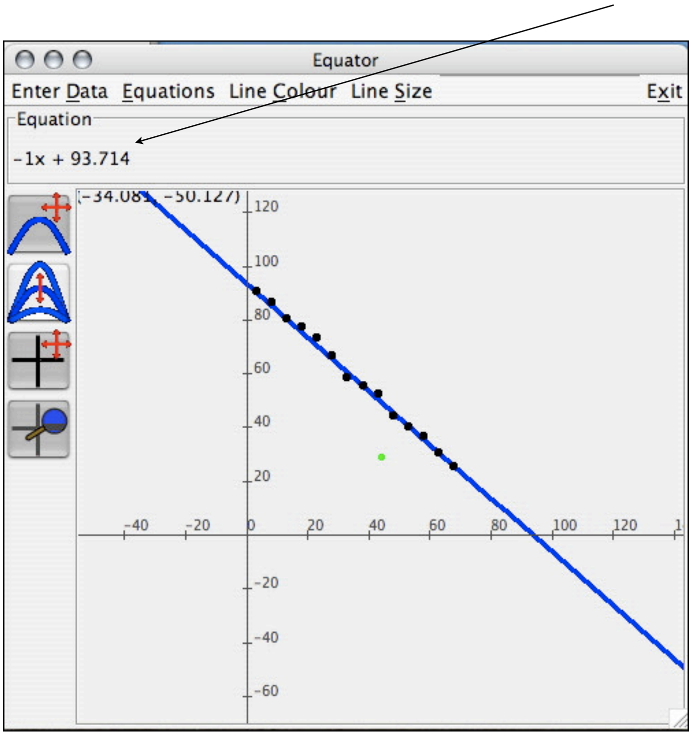
A simple procedure for 'finding' the data is:

- **Click the scroll button.**
- **Drag the axes up to about 80 (because the pizza starts at about 80 degrees C.**
- **Click the Zoom button.**
- **Click in any white space to Zoom out.**
- **You should now see the data and be ready to zoom in to get a better view.**



Fitting the function

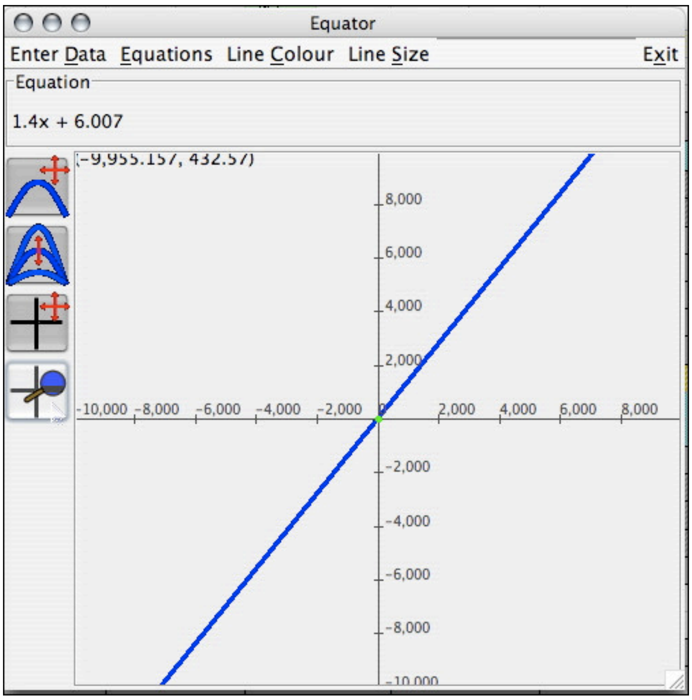
The aim of the program is to be able to manipulate the graph so that students can get a fit by



eye. In the image here, the straight line appears a reasonably good fit to the data, and the function is reported as -1x + 93.714

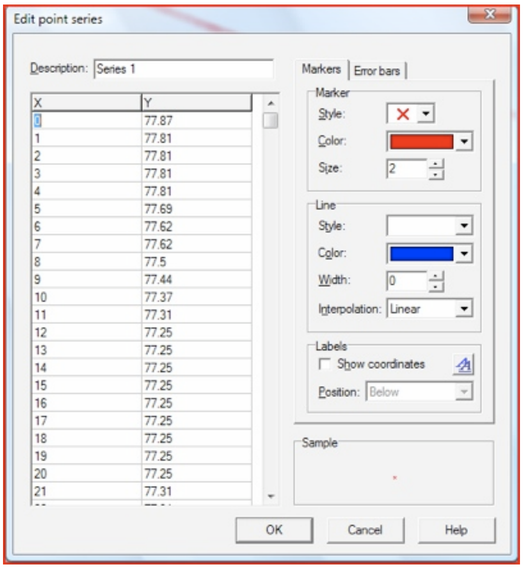
Zooming

To zoom out from the graph you have to click on the magnifying glass and then click on the graph at the centre of the enlargement.

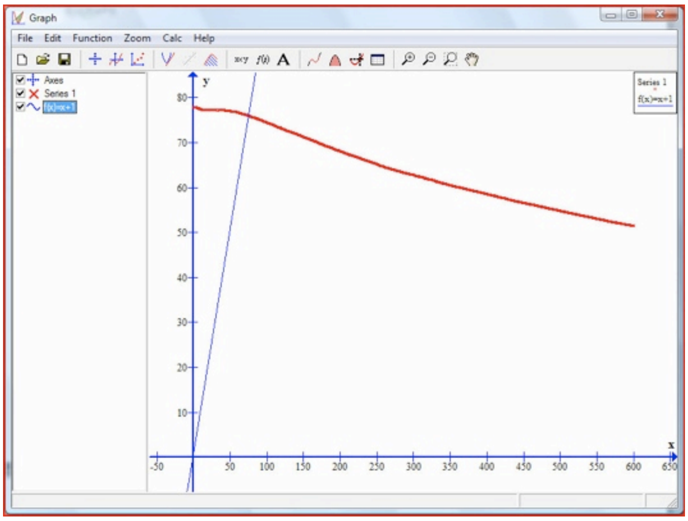


Graph

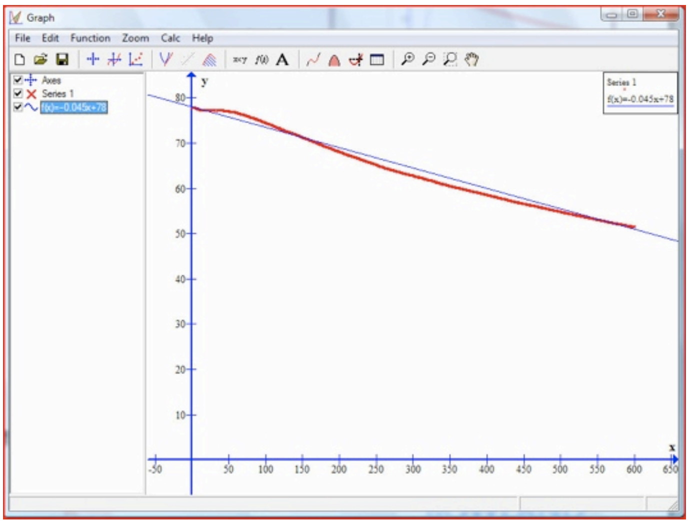
This is open-source software (i.e. free to use) from http://www.padowan.dk/graph/
You can copy and paste a complete set of data from each pizza experiment.
Copy only the number values from both columns of the data table of values in your data logging software (e.g. Logger Lite, Data Harvest etc.).
Launch Graph and click **Function/Insert Point** Series
Use **ctrl + V** to paste the values.
Select a cross of size 2 for your **Marker**.
Click **OK**



Now click Function/Insert Function and type a function to try to fit the points. Type your function into the Function Equation f(x)= box. Note DO NOT type y = or f(x) = into the box.
In the lesson notes, we suggest you start with a linear model. Students may suggest something like f(x)=x+1



This leads into a discussion about how the function can be changed to fit the curve. Double click on the function in the key on the left hand side of the screen to amend the function.
When a good fit has been found, the function can be used to calculate a) the temperature at different points where data has been collected, and b) the temperature at different point in the future, to see if it provides sensible predictions.



Logger Lite Data Logging Software

Logger Lite is supplied with Go Temp probes. These plug into the USB port of a windows computer.

It is necessary to pre-set the length of the experiment. In our case 10 minutes worth of readings are sufficient. Hence, click **Experiment/DataCollection** … and set the length to 600 seconds.

Click **OK** to confirm.

Prepare the experiment.

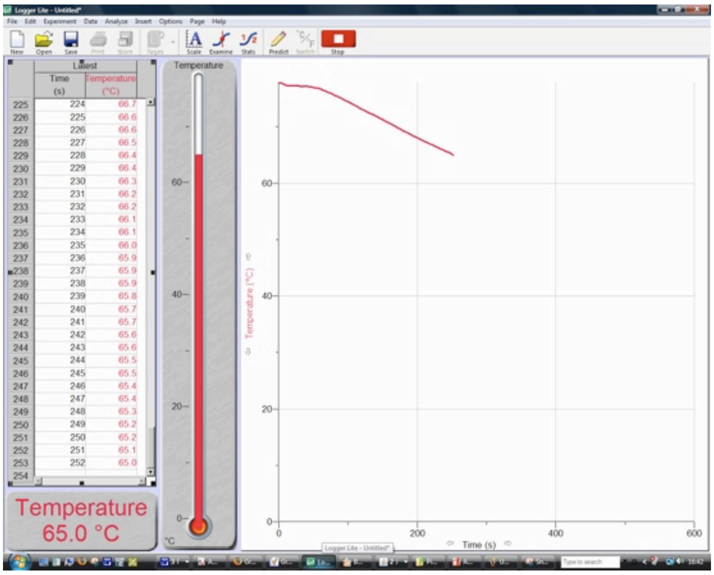
Push the probe into the hot pizza (into the topping is better than into the base).

Click the green **GO** button.

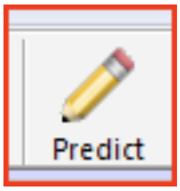
The experiment will end after 600 seconds.

Save your experiment when you are finished.

Click **File/New** to start a new experiment.



Note: For the next experiment it is necessary to set the length to 600 seconds again.



Note that *Logger Lite* has a nice facility for predicting as the data is being collected. Click the **Predict** button and draw a line extending the existing data. This creates a new column of predicted data. This can be compared with the actual data over the next period.

Appendix F: FAQs

This does not look like a 3-part lesson to me. How can I run a lesson without an aural and mental starter and a plenary?

Lessons should be arranged as 'fit for purpose' rather than a 'one size fits all'. With this kind of lesson we are looking for students to think about the mathematics to which they are being introduced.

I am worried that students will be restless while they are waiting for the pizza to cool, what should I do?

As the pizza cools, students should be predicting the temperature at different point in the future e.g. In the next minute, after ten minutes, after and hour etc. In our experience the students are very engaged to see if they can 'guess' correctly and will develop increasingly sophisticated thinking to get better at it.

How can lower ability students deal with this kind of mathematics?

These lessons have been trialled in ordinary schools with the full range of ability and students of all abilities have enjoyed the lessons. The objective of the lessons is not to teach the mathematics not that students understand all the concepts that they encounter. What is important is that students understand that mathematics can be used to help make sense of real problems. It is particularly pertinent to lower attaining students that simple models are generally better than more complex ones.

I can't get *Equator* to work.

This is a little Java script program which enables limited curve fitting by 'dragging'. The most likely cause of any problems is that your Java script program is not up to date. To solve this go to the Sun Microsystems Java site http://www.java.com/en/download/index.jsp or Google 'Java' and follow the links.

If Equator is running and you have used data from an experiment or from the files supplied and you cannot see the data points, this is because you will need to find the data. Carefully follow the procedure suggested in the blue box on page 34.