

CASE OVERVIEW

IN THIS CASE...

- Pupils play the roles of designers of mascots for a new school. The mascot character will reside on a new handheld learning/entertainment device for 5 – 7 year olds – called **My Q**. You, the teacher, are their supervisor. You have the opportunity to dramatise your role.
- Pupils set design criteria, draw with pixels, estimate area, and resize for different size displays on the device.
- Main mathematics: area as square units; calculating by counting, decomposition and formulae; proportional and non-proportional resizing; changing ratios of sides of rectangles and the effect on area.

CASE FEATURES

4 lessons with extensions; provides a starter experience with cases.

Pupils are immersed in a design experience.

A set of presentation slides guides lessons; the presentation is a script from which the teacher may improvise.

The teachers' questions and guidance help make the mathematics more explicit.

There are opportunities for mathematical calculation, reasoning, writing and discussion.

Ideas for extensions include survey design and investigation of tessellations.

Lessons could be shared with a Design teacher.

MATERIALS

Provided: Presentation for **Design The Mascot**

Pupil handouts

Digital Drawing Tool Flash-based application

Required: Computers, at least one for every 2-3 pupils

Computer with large display or electronic white board for the teacher

Colour pencils or markers, graph paper, calculators

Optional: Examples of characters that appeal to young children

Websites that focus on digital design and use pixels

KEY STAGE 3 MATHEMATICS CURRICULUM

This case provides substantial opportunities for pupils to work with enlargements:

Making and using enlargements of 2-D shapes, with positive scale factors — whole numbers and fractions.

Understanding the implications of enlargement for area.

The entire case is an extended exercise in problem solving, communication and reasoning in the area of “shape, space and measures”. Specific skills include

Identifying given information and what further information is needed

Using geometric diagrams to communicate mathematically

Exam papers in the past have included squares and rectangles with the same area and proportional enlargements.

<i>LESSON</i>	<i>DESIGN THEME</i>	<i>MATHEMATICS THEME</i>
1	Designing	Decomposing and other area strategies
2	New Displays	Proportional and non-proportional enlargements — including effects on area.
3	Simple Shapes	Area approximation.
4	New Display Formats	Proportional and non-proportional enlargements — including effects on area.

LESSON NOTES

Lesson 1



Slides 1 – 9

Part 1

Design Criteria

Your Q School MASCOT design must

- **Appeal to the Q School's 5 - 7 year old pupils.**
- **Cover at least 60% of the display area.**

Design THE MASCOT with Pixels

Slides 10 – 11: Part 1 Handout

- Get into the spirit of role-play—pupils play designers. You are their supervisor. Read dramatically! Improvise!
- You or your pupils may bring in images of handheld electronic devices or toys with electronic displays—or the actual devices. Examples of handhelds include mobile phones and PDAs. Toys with electronic displays include the Bandai Tamagotchi, the Nintendo Game Boy, the Nintendo DS, and the Sony PSP. Each of these devices displays pixel based art including characters.
- You or your pupils may bring in examples of school mascot designs and graphics.
- Optional: Demonstrate DigiTool to show how pupils will eventually make their design.

- Briefly discuss 60%: a bit more than one half.
- Elicit pupils' knowledge of young children.
- Pupils should respond to the prompts in yellow (see slide 11 for an example).
- Possible criteria: bright colours; big face; like known characters on television.
- Compare two designer responses: different "focal" children lead to different design requirements and ideas.
- Optional: You may tell pupils to use a geometric shape as part of the design, or perhaps that the mascot must cover less than 80% of the screen area as well as more than 60%.

Part 1

Rapid Sketching

✓ **Make 2 sketches of mascots.**

Be creative but...

keep the design criteria in mind.

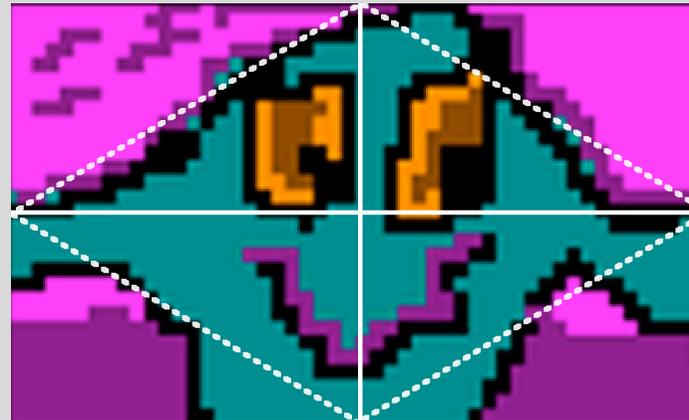
✓ **Estimate how much of the display area is occupied by each **MASCOT**.**

✓ **Explain how each **MASCOT** design satisfies (or doesn't) the criteria.**

Design THE MASCOT with Pencil

Slide 12: Part 1 Handout

- Pupils use the spaces on the handout, which are the size of the **My Q** display.
- No grid is provided to encourage freehand drawing and to discourage counting for estimation of area.
- Continue to encourage creativity. Keep the pace *rapid*: don't let pupils get bogged down in making "perfect" designs.
- Estimation techniques may rely on pupils de-composing area. Decomposing area is a strategy for solving problems on previous Key Stage 3 exam papers.
- Possible estimation technique (see image below): The four triangles in the centre of the rectangle contain most of the character, but the character is a bit bigger, considering how much of its area is outside the triangles. The four triangles have area equal to 50% of the whole rectangular display. This is because the area of each right angled triangle is half the area of the rectangle in which it is inscribed. Therefore, the character's area is about 60% of the display.



<p>Part 1</p> <h2>Finalize Your Design</h2> <ul style="list-style-type: none">✓ Use the DigiTool to make a pixel-by-pixel final design. Use your sketches to guide you.✓ Explain how your design meets each of the criteria. <p><small>Design THE MASCOT with Pixels</small></p>	<ul style="list-style-type: none">▪ Pupils use the Digital Drawing Tool to create a final design, using pixels. It should incorporate the best of the two rapid sketches.▪ Pupils should, in writing, explain how they know their MASCOT takes up at least 60% of the display area.▪ Possible methods: The DigiTool grid is 50 by 30 pixels. Now, pixel counting or estimating can be compared with their previous method. Pupils may use knowledge of geometric shape formulae: for example, circles.▪ You may review or re-teach concepts or formulae. Finding areas by counting squares is pitched at Level 4 in the National Curriculum.▪ Pupils should print or take a screenshot of their MASCOT design. They may wish to refer back to this image in the next lesson.
<p>Slide 13: Part 1 Handout</p> <p>Part 1</p> <h2>Discuss the Maths</h2> <ul style="list-style-type: none">• How did you estimate the area covered by your MASCOT ?• What makes a good MASCOT design? <p><small>Design THE MASCOT with Pixels</small></p>	<ul style="list-style-type: none">▪ This discussion ensures that pupils are exposed to different methods for finding the percentage of the area of the display that is covered by the MASCOT.▪ The second question is less mathematical in nature, but some pupils may comment on shapes and sizes that are appropriate; look for mathematics as well as design insights.▪ Make sure that pupils print or take a screenshot of their MASCOT. There is no way to save their design in the DigiTool.

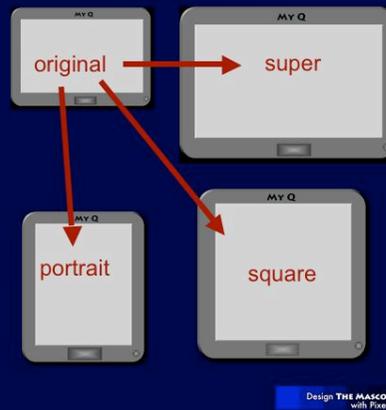
Lesson 2

Part 2

New Devices, Different Size Displays

Three new **MY Q** devices will use different size displays.

Your job is to investigate how the new display sizes will affect your **MASCOT** design.



Slide 15-16

- Have pupils watch the video from slide 15 (this is the second client message)
- As supervisor, make sure that the pupils understand that there are three new proposed models of the My Q device. Ask pupils to describe each one in relation to the others, using mathematics.
- Tell pupils they will investigate the new displays, mathematically, in relation to the original My Q's display.
- In the end, they may choose any of the new My Qs as best for their **MASCOT**.

Part 2

New Devices, Different Size Displays

✓ Predict the new sizes and fill in the table.

- Don't forget to use the specified width to height ratios and meet the pixel requirements

Design THE MASCOT with Pixels

Slide 17: Part 2 Handout

- Pupils use the given information in the table and their knowledge of the current display (50 by 30 pixels) to complete the table.
- The ratio between the sides of the display is often called the “aspect ratio.” There are different conventions for which number comes first in the ratio. We put the width first.
- Displaying an image on a device with a different aspect ratio is commonly accomplished through cropping; otherwise the image may be distorted. For example, showing an image made for regular television (a 4:3 aspect ratio) on a high-definition television (a 16:9 aspect ratio) will either not fill the screen or result in a distorted image.
- Answers will vary in the table, but must meet the constraints. Many pupils will double the dimensions of the given example of the **SUPER MY Q**, which is much larger than needed but still correct.

Handheld	width (in pixels)	height (in pixels)	ratio of width to height	pixels
My Q (original)	50	30	5:3	exactly 1500
Super My Q	75	45	5:3	3375
Square My Q	39	39	1:1	1521
Portrait My Q	30	50	3:5	exactly 1500

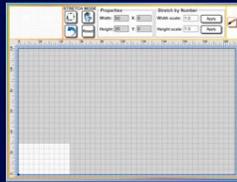
greater than 3000

about 1500

Part 2

Resizing Your **MASCOT**

✓ For each new display, use the DigiTool, in Stretch Mode. Resize your shape in two ways. Check your table predictions, and revise as necessary.



Slide 18 – 19: Part 2 Handout

- Now pupils can test the predictions in their tables using the DigiTool in Stretch Mode. Instead of taking the time to redraw their **MASCOT**, pupils will choose and draw a geometric shape, which they will use for their tests. They may wish to recreate the outline or silhouette of their **MASCOT**. Slide 19 explains how to use the DigiTool in two ways, as asked for in the prompt. Preview the questions they will need to answer by using slides 20 and 21.
- In using the Stretch Mode to resize the display, pupils will:
 - Use the Properties mode to test their table numbers. Does the rectangle look like the new display: Portrait, Super or Square? If not, where was the mistake? Revise if needed.
 - See the effect of the resizing on their shape: Is it distorted? How? How do they think that their **MASCOT** would look? Would they like the look of the distorted designs? (Some distorted images may be more appealing than the original!)
 - Use the stretch mode to investigate the scale factors for resizing, which are not in the table. Ask pupils to make notes on these on their handout. They need to find scale factors that give the same dimensions as in their revised table.
- Observations to invoke:
 - They should find that the Super display is an enlargement of the original with the same factor for height and width. This keeps the ratio between the sides as 5:3, as required.
 - The other three cannot be re-sized by the same factor for width and height.

Part 2

Discuss the Maths

- How did you decide on the width and height for the **SQUARE MY Q**?
- How did you use numbers to resize the display for the **SUPER MY Q**?
- How did you use numbers to resize the display for the **PORTRAIT MY Q**?
- Which new display will be the best for displaying your **MASCOT**?

Design THE MASCOT with Pixels

Slide 20: Class Discussion

- The width and height of **Square My Q** must be the same and the total pixels must be close to 1500, which is not a perfect square. Pupils may begin at 40 (square root of 1600) and test numbers near it until the square is near 1500. Pupils may find that 30 squared is 900 and deduce that their answer should be closer to 40 than 30.
- Introduce or review proportional enlargement, similar figures, and scale factor. Each **Super My Q** is a proportional enlargement of the original **My Q**, because the ratio of the sides is still 5:3. Therefore, pupils have to use the same scale factor for width and height.
- The ratios for the original and portrait are inverses: 3:5 and 5:3. The height and width are simply reversed. More advanced pupils may find that the factors for re-sizing to portrait size, 0.6 and 1.666..., are multiplicative inverses.

Many **SUPER MY QS**

Name	Width	Height	Width Scale	Height Scale	Total Pixels
original	50	30	1	1	1500

Design THE MASCOT with Pixels

Slide 21: Class Discussion

- Fill in this table, using different pupils' **Super My Q** answers. Choose those with greatly different scale factors. Choose a fractional scale factor if possible.

Part 2

Discuss the Maths

- Find as many relationships as you can among width, height, and scale factors of the **SUPER MY Qs**.
- What is the relationship between the width, height and number of pixels for each row in the table?
- How do scale factors for width and length affect the number of pixels needed for each **SUPER MY Q**?

Design THE MASCOT
with Pixels

Slide 22: Class Discussion

- Relationships in the table:
 - The width and height are always in ratio 5:3. Because of this, the scale factor for width and height is always the same, but there are different scales for different **Super My Qs**.
 - The larger the scale factor, the larger the width and height.
 - Scale factors need not be whole numbers. If no pupil has done so, try a scale factor of 1.5, which gets close to the area of 3000.
- The number of pixels that make up the whole design is just the area of the display, expressed in pixels.
- For **Super My Qs** with a scale factor of 2, the area is increased by a factor of 4.
- Elicit the general principle that if a rectangle is enlarged by a factor of n , the area is increased by a factor of n^2 .
- Optional extension: Investigate the effect of scale factors on area for **Square My Q** as compared to the original.

Lesson 3

Part 3

Circles and Triangles

Do They Meet the Criteria?

The Q School's steering committee wants to investigate making the **MASCOT** designs simple geometric shapes like circles and triangles.

Would designs of circles or triangles meet the design criteria?



Design THE MASCOT with Pixels

Slide 23-24

- Have pupils watch the video from slide 23 (this is the third client message).
- As supervisor, make sure that the pupils understand that the proposal is to use only circle or triangle-shaped mascots.
- The pupils will investigate mathematically whether circle or triangle-shaped mascots meet the design criterion of covering 60% of the display area.
- Encourage the pupils to justify their answers.

Part 3

Sizing Up a Circle

✓ What is the approximate area, in pixels, of the largest circle that can fit the original **MY Q** screen?

✓ If your **MASCOT** were shaped as a circle, would it cover at least 60% of the **MY Q** screen?

Design THE MASCOT with Pixels

Slide 25: Part 3 Handout

- Pupils work through the same exercise for both the circle and the triangle.
- A typical pupil work process may be:
 - Determine maximum dimensions of the shape (for example, 30 pixel diameter for the circle);
 - Calculate the shape's area using the maximum dimensions;
 - Calculate the shape's area as a percentage of the display area.

Part 3

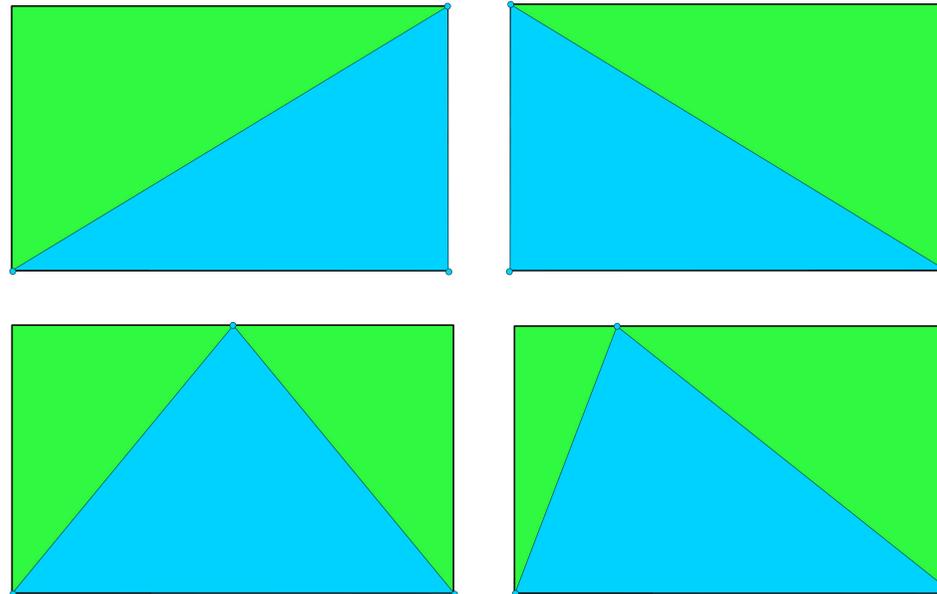
Sizing Up a Triangles

- ✓ What is the approximate area, in pixels, of the largest triangle that can fit the original **MY Q** screen?
- ✓ If your **MASCOT** were shaped as a triangle, would it cover at least 60% of the **MY Q** screen?

Design THE MASCOT with Pixels

Slide 26: Part 3 Handout

- For triangles, working out the maximum area may require consideration of various kinds of triangles.
- Here is one way to answer: One way to maximize the area of a triangle is to have two of its vertices the same as two of the vertices of the rectangle (see below for examples). Consider, for example, a right angled triangle that “cuts” the display in half with its hypotenuse. Its area is 50% of the rectangle’s area. Any other maximal triangle will have the same area, with heights of 30 and widths of 50, like the displays. Therefore, the most area that a triangle can “occupy” is 50% of the display.



Part 3

Discuss the Maths

- How did you decide on the diameter for the largest circle that could fit in the original My Q screen?
- How did you calculate what percentage of the screen that the largest circle would cover?
- How did you decide on the diameter for the largest triangle that could fit in the original My Q screen?
- How did you calculate what percentage of the screen that the largest triangle would cover?

Design THE MASCOT
with Triads

Slide 27: Class Discussion

- Discuss the process the pupils used to answer the questions on their handouts.
- Make sure pupils justify their answers, and check for understanding.

Lesson 4

Part 4

T-shirts, Coffee Mugs, What's Next More Displays

The Q School's steering committee wants the **MASCOT** on many different items.




How will you change your design?

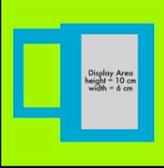
Design THE MASCOT with Pixels

Slide 28-29

Part 4

Resizing Your **MASCOT**

- ✓ For each new format, use the DigiTool, in Stretch Mode. Resize your shape as necessary.
- ✓ Explain how your Mascot design would need to change to meet the design criteria for each object.

Design THE MASCOT with Pixels

Slide 30: Part 4 Handout

- Have pupils watch the video from slide 28 (this is the fourth client message).
- As supervisor, make sure that the pupils understand that the proposal is to put their **MASCOT** designs on other objects like t-shirts and coffee mugs.
- This activity is similar to the activity in part 2. Pupils investigate the revisions necessary for their mascots to meet the design criterion of covering 60% of the new display areas.
- Encourage the pupils to justify their answers.
- Pupils use the given information in the table and their knowledge of the current display (50 by 30 pixels) to complete the table.
- As in Part 2, instead of taking the time to redraw their **MASCOT**, pupils will choose and draw a geometric shape which they will use for their tests. They may wish to recreate the outline or silhouette of their **MASCOT**.
- In using the Stretch Mode to resize the display, pupils will:
 - Use the Properties mode to test their table numbers. Does the rectangle look like the display area of the t-shirt / coffee mug? If not, where was the mistake? Revise if needed.
 - See the effect of the resizing on their shape: Is it distorted? How? How do they think that their **MASCOT** would look? Would they like the look of the distorted design?
- Observations to invoke:
 - Pupils should find that the t-shirt display is not an aspect ratio that they have previously encountered.
 - Pupils may recognize that the aspect ratio of the coffee mug matches that of the Portrait My Q.

Part 4

Discuss the Maths

- How did you decide on the width and height for your **MASCOT** for display on a t-shirt?
- How did you decide on the width and height for your **MASCOT** for display on a coffee mug?
- What would your process be for investigating the redesigns necessary for displaying your **MASCOT** on other kinds of objects?

Design THE MASCOT
with Pixels

Slide 31: Class Discussion

- Discuss the process the pupils used to answer the questions on their handouts.
- Make sure pupils justify their answers, and check for understanding.

Thank you for your hard
work.

How will you use the maths
you have just learned not
only in school but in the rest
of your life?

Design THE MASCOT
with Pixels

Slide 32: Class Discussion

- This is your last opportunity to play supervisor. Thank pupils for a job well done and choose particular strengths to note: a wide variety of designs, astute mathematical observations. You may ask pupils to state some project work of which they are particularly proud.
- Pupils may discuss photo manipulation software and scale drawings. You can add that digital graphics work is, in fact, a growing field for real careers.

Ideas for Extensions	<ul style="list-style-type: none">▪ Part 1: Pupils could design, conduct, and analyze a survey of 5-7 year olds to identify their visual design preferences (shapes, colours, etc.).▪ Part 2: Investigate the effect of scale factors on area for Square My Q as compared with the original.▪ Part 3: Pupils investigate use of circle-shaped mascots with rectangular displays with various aspect ratios and determine whether a circle-shaped mascot could ever meet the design criterion of covering 60% of a rectangular display area. Pupils could also find the aspect ratio that would result in an area coverage of approximately 60%.▪ Part 4: Pupils could investigate the use of tessellations to solve design problems including the use of mascot design for irregularly shaped objects (for example, an archway) or objects with more extreme width to height ratios, such as scarves.▪ Part 4: Pupils could use t-shirt and product customization websites such as zazzle.com or cafepress.co.uk to translate their designs into real objects.
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