Lesson 3: Cows gone wild

Narrative

Mavis's famous cavorting cattle have kicked down the gate of their shed, and are wreaking havoc across the festival site, leaving a trail of splintered wood, broken glass and hoof prints. Now the cows have attacked the painstakingly created main stage, reducing it to a pile of planks and rubble. Using only materials found on the site – drink cans, hay bales, crates, and so on – the FSS must build a new 'green stage' in time for the big show and headline act.

Problems

Main challenge

The floor of the new 'green stage' will be rectangular. It will be built using a straight edge of the field as one side, and will be supported on a single layer of hay bales. However, the teams have only 100 m of lighting strips to go along the other three edges of the stage. What advice should they give Mavis about the size of the stage floor to make its area as large as possible? Can the teams prove that they have suggested the largest possible area? How many hay bales will be needed? How should they be positioned to use as many as possible?

Alternative problems or homework

1 The squad teams intend to use 1000 empty cans to build a scale model of a stage in the shape of a cuboid with height (h), depth (d) and width (w), subject to the condition that $h \leq d \leq w$. They investigate the different models they could make and select the one that they will use. They work out the size of their real stage when they replace each can with a plastic crate.

2 A triangular stage has been built across a right-angled corner of a field but it needs to be made higher. The teams must work out how to use hay bales to raise the height of the stage floor by 1.2 metres.

Extension problem

3 A new 'green stage' is to be built using the middle of one side of the field as one side of the triangle. Two lighting strips are available. One is 30 m long and one is 40 m long. These will form the other two sides of the triangle. What is the maximum possible area of the stage?

Skills required

In this lesson pupils will need to:

- solve problems involving measurements in a variety of contexts
- interpret and make scale drawings
- use formulae for the area and perimeter of a rectangle, the area of a triangle and the volume of a cuboid and right prism
- make and justify estimates and approximations of calculations
- use a calculator effectively and efficiently to carry out more difficult calculations, interpreting the display in different contexts, including measures
- explore the effect of varying values
- use algebraic methods to solve problems, using ICT as appropriate.

Torbury resources

3.1 Video clip: (13”**)** Clip from a mobile phone of cows escaping

3.2 Audio clip: (29”**)** Phone call from Mavis Broom to the FSS to introduce the challenge

3.3 Slides presenting the main challenge, with a graphic of the hay bales and their dimensions

3.4 A4 resource sheet with graphics of the hay bales, crates and cans, and alternative or further problems for pupils (if you are using one of the problems for homework, print one sheet per pupil; if no homework is being set, print one per pair)

3.5 Video clip: (27”**)** Revellers are gathering for the big show. There is a glimpse of a stage with spot lights. The headlining band is spotted arriving on site. The festival goers are eagerly awaiting their performance later on.

Other resources

Scientific calculators, squared paper, stiff card, rulers; for the extension problem, graph paper or graphics calculators, a function graph plotter

Main activity

Introduce the next day of the festival. Play **Resource 3.1**, a video clip of cows escaping and wreaking havoc (13”).

Play **Resource 3.2**, an audio clip of a phone call from Mavis to the squad explaining the challenge (29”). Back this up by presenting the challenge using slides 1 to 3 of **Resource 3.3**, then set the teams to work.

Differentiation

The first two of the further problems or homework on **Resource 3.4** are easier. The third is more challenging and, to prove the result, requires some knowledge of trigonometry. For this third problem you may need to prompt pupils to consider different types of triangles.

Review

Bring the class together to discuss and compare approaches to the problems and solutions, and to justify their decisions. If you want to discuss graphical solutions, you could either use your school’s function graph plotter, or you could download and use a free one, e.g. Graph, which you can find at **www.padowan.dk/graph/.**

To finish, play **Resource 3.5**, a video clip (27”). The revellers are gathering for the big show. There is a glimpse of the stage with spotlights. The headlining band is spotted arriving on site.

Optional homework

If you are setting homework, ask pupils to do a remaining problem from **Resource 3.4** (or a remaining problem from Lessons 1 or 2).

Solutions for Lesson 3

Main challenge (Resource 3.3)

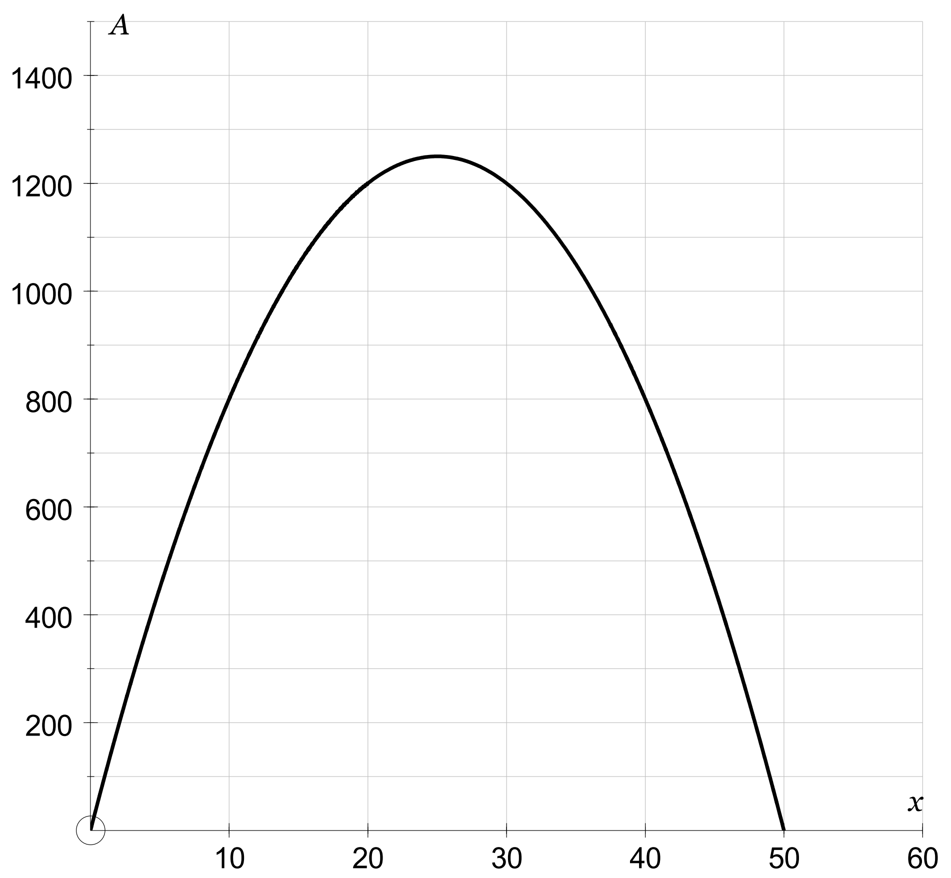
The problem can be solved by experimenting by drawing on squared paper, giving the long side of the rectangle against the wall (length 50 m, width 25 m), giving an enclosed area of 1250 m^2.
2500 hay bales standing on a 1 metre by 0.5 metre face could be fitted below this maximum area, creating a stage 0.5 metre high.

Alternatively, 5000 hay bales standing on a 0.5 metre by 0.5 metre face could be used, creating a stage which is 1 metre high, which would be the preferred option.

Pupils can prove the result for the maximum stage area algebraically.
If the rectangle has width x and length y, then: $y + 2x = 100$
The area A is given by: $A = yx = 100x - 2x^2$

There are various approaches to solving this equation.

The first approach is to draw the graph of $A = 100x - 2x^2$, either using graph paper or using a graphics calculator, and show that the maximum value of A is 1250, which occurs when $x = 25$.



The second approach is using trial and improvement, homing in on $x = 25$, $A = 1250$.

The third approach is to complete the square, which gives: $A = 2(50x - x^2) = 2(625 - (25 - x)^2)$.
This has a maximum value when: $25 - x = 0$, or $x = 25$.
Substituting gives $y = 50$ and $A = 1250$.

Further problems or homework (Resource 3.4)

1 a There are 19 possible models for height × width × length, given height ≤ width ≤ length.

height 1 can	height 2 cans	height 4 cans	height 5 cans	height 10 cans
1 × 1 × 1000 1 × 2 × 500 1 × 4 × 250 1 × 5 × 200 1 × 8 × 125 1 × 10 × 100 1 × 20 × 50 1 × 25 × 40	2 × 2 × 250 2 × 4 × 125 2 × 5 × 100 2 × 10 × 50 2 × 20 × 25	4 × 5 × 50 4 × 10 × 25	5 × 5 × 40 5 × 8 × 25 5 × 10 × 20	10 × 10 × 10

b Pupils' choices and reasons may include factors such as making sure that the height of the stage is sufficient for the audience to see the performers, and making sure that the stage floor has sufficient length and width.

c Answers will depend on the choices made in part **b** but, for every choice, there are six possibilities when the crates are substituted, depending on which way round the crates are positioned. For example, if a team has chosen 2 × 10 × 50 cans as the choice of stage, then the actual stage made from crates could be:

height	width	length
2 × 0.3 = 0.6 m	10 × 0.4 = 4 m	50 × 0.6 = 30 m
2 × 0.3 = 0.6 m	10 × 0.6 = 6 m	50 × 0.4 = 20 m
2 × 0.4 = 0.8 m	10 × 0.3 = 3 m	50 × 0.6 = 30 m
2 × 0.4 = 0.8 m	10 × 0.6 = 6 m	50 × 0.3 = 15 m
2 × 0.6 = 1.2 m	10 × 0.3 = 3 m	50 × 0.4 = 20 m
2 × 0.6 = 1.2 m	10 × 0.4 = 4 m	50 × 0.3 = 15 m

2 The hay bales are used so that the 1.2 m edge forms the height of the extension.
The square base of 0.5 m by 0.5 m is used to cover the triangle as far as possible.
69 bales will fit in a row along the base of the triangle.
There will be 69 rows altogether, with one bale fewer in each row.
Altogether there will be 1 + 2 + 3 + 4 + … + 69 = $\frac{1}{2}$(69 × 70) = 2415 bales.

Teams who are not familiar with the formula for the nth triangular number could draw a diagram on squared paper, using 1 cm to represent 0.5 m.

They could also work it out like this:

The sum S of the first 69 natural numbers is: S = 1 + 2 + 3 + … + 69
Writing the sum in reverse order is: S = 69 + 68 + 67 + … + 1
Adding gives: 2S = 70 + 70 + 70 + … + 70 = 69 × 70
So dividing by 2 gives: S = $\frac{1}{2}$(69 × 70)

Extension problem for the most able pupils

3 Teams could estimate the result using squared paper and use ruler and compasses to construct the triangle for different lengths of the side along the side of the field. The maximum area is formed when the angle between the two lengths of fence is a right angle.

Area A is $\frac{1}{2}$ × 30 × 40 = 600 m2^2

To prove the result, consider two lengths of fence a and b, with an angle x between them. The area of the triangle formed is $\frac{1}{2}$ ab sin x, which has a maximum value when sin x = 1, i.e. x = 90°.